1.

a. ((¬ p) → q) ↔ ((¬ r) ∧ p)

b. p ↔ ((¬ p) → (q → r))

2.

a. Show that ((p ↑ q) ↑ r) ↔ ((p ↑ (q ↑ r)) is satisfiable. We need to show that there is an interpretation I such that ((p ↑ q) ↑ r) ↔ ((p ↑ (q ↑ r)) = T. So, we must have((p ↑ q) ↑ r) = T and ((p ↑ (q ↑ r)) = T which means we must have *I*(p) = T, *I*(q) = T, and *I*(r) = T. Hence, ((p ↑ q) ↑ r) ↔ ((p ↑ (q ↑r)) is satisfiable because (q) = T, and (r) = T will satisfy A.

Show that ((p ↑ q) ↑ r) ↔ ((p ↑ (q ↑ r)) is falsifiable. We need to show that there is an interpretation I such that ((p ↑ q) ↑ r) ↔ ((p ↑ (q ↑ r)) = F. So, we must have ((p ↑ q) ↑ r) = F and ((p ↑ (q ↑ r)) = T which means we must have *I*(p) = F, *I*(q) = T, and *I*(r) = T. Hence, ((p ↑ q) ↑ r) ↔ ((p ↑ (q ↑r)) is falsifiable because (q) = T, and (r) = T will falsify A.

b. Show that (p ↔ q) → (p ↔ (q ↔ p)) is satisfiable. We need to show that there is an interpretation I such that (p ↔ q) → (p ↔ (q ↔ p)) = T. So, we must have (p ↔ q) = T and (p ↔ (q ↔ p)) = T which means we must have (p) = T and (q) = T. Hence, (p ↔ q) → (p ↔ (q ↔ p)) is satisfiable because (q) = T will satisfy A.

Show that (p ↔ q) → (p ↔ (q ↔ p)) is falsifiable. We need to show that there is an interpretation I such that (p ↔ q) → (p ↔ (q ↔ p)) = F. So, we must have (p ↔ q) = T and (p ↔ (q ↔ p)) = F which means we must have (p) = F and (q) = F. Hence, (p ↔ q) → (p ↔ (q ↔ p)) is falsifiable because (q) = F will falsify A.

3.

a. Show that ((p ∧ q) → r) → ((p → r) ∨ (q → r)) is valid. BWOC assume that A is not valid. That is, there is an interpretation I that falsifies A. So, ((p ∧ q) → r) → ((p → r) ∨ (q → r)) = F which means that ((p ∧ q) → r) = T and ((p → r) ∨ (q → r)) = F. But if ((p ∧ q) → r) = T, then ((p ∧ q) → r) → ((p → r) ∨ (q → r)) = T. So, our assumption is wrong. Hence, = T for all interpretation I. That is, A is valid.

b. Show that (p← (q ← r)) → ((p ∧ q) → q) is valid. BWOC assume that A is not valid. That is, there is an interpretation I that falsifies A. So, (p← (q ← r)) → ((p ∧ q) → q) = F which means that (p← (q ← r)) = T and ((p ∧ q) → q) = F. But if (p← (q ← r)) = T, then ((p ∧ q) → q) = T. So, our assumption is wrong. Hence, = T for all interpretation I. That is, A is valid.